F09 Electrical conductivity

of metals and semiconductors

Basic knowledge:

- Boltzmann and Fermi-Dirac-statistics
- Band structure for metals, undoped and doped semiconductors
- Basic models of temperature dependence of electrical resistivity in metals and semiconductors
- Type 1 superconductors
- Four-point-measurement technique

Literature:

[1] Ibach/Lüth:	„Festkörperphysik“, Springer-Verlag, Berlin (1981) Kapitel 5,6,7,8,10, Supraleitung
[2] Gerthsen:	„Physik“
[3] Ch. Kittel:	„Festkörperphysik“, Oldenbourg-Verlag (1988)

Content:

1. Electrical conductivity

1.1 Metals

Using the Drude model and assuming a free electron gas, the electrical conductivity can be expressed as:

$$\sigma = \frac{ne^2\tau}{m^*}, \tag{1}$$

where τ denotes the relaxation time (the average time between two collisions), n the charge carrier density, and m^* the effective electron mass. Since the n is nearly temperature-independent due to the Fermi distribution, the temperature dependence of σ arises primarily from τ. Two main scattering mechanisms determine the relaxation time:

1) *Impurities*: Scattering from vacancies or impurities is temperature-independent, leading to a residual resistivity ρ_{ST} at low temperatures.

2) *Phonons*: At higher temperatures, thermally excited lattice vibrations (phonons) scatter conduction electrons. An important characteristic temperature is the Debye temperature Θ_D. Below the Debye temperature, with increasing temperature additional phonon modes with shorter wavelengths (higher energy) become accessible. Above the Debye temperature, the phonons with the shortest possible wavelength (e.g. vibrations between two neighboring atoms) are occupied, so no additional modes become available with increasing temperature. As a result, two temperature regimes emerge in the resistivity:

   $T \ll \Theta_D$: $\rho_{Ph} \propto T^5$ for simple metals

   and

   $T \gg \Theta_D$: $\rho_{Ph} \propto T$.

The total resistivity is given by the Matthiessen rule, a typical curve is sketched in Fig. 1:

$$\rho = \rho_{ST} + \rho_{Ph}. \tag{2}$$

1.1.1 Superconductivity

Some metals show superconductivity at low temperatures. They will show the same regimes as the ones discussed above. However, below a critical temperature T_C, the resistivity vanishes. Typical values of T_C are:

Tantalum (Ta):	4,48 K
Lead (Pb):	7,20 K

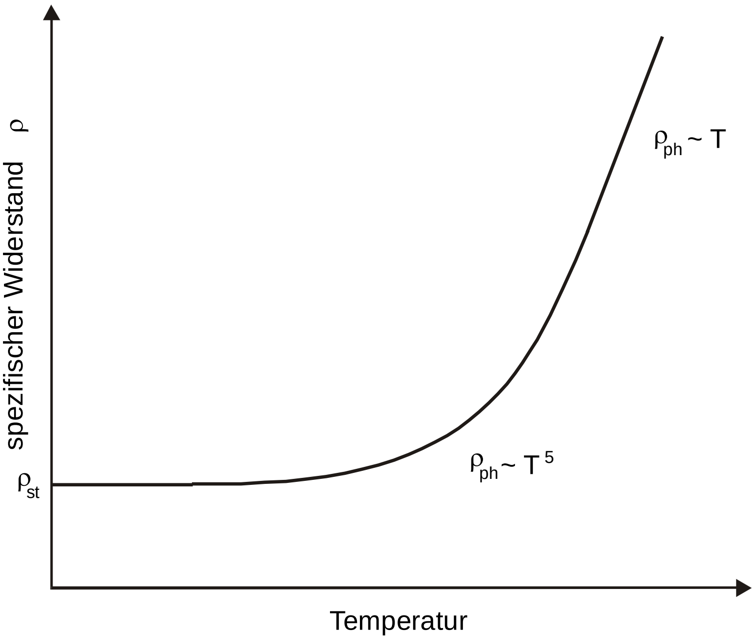

Figure 1: Temperature dependence of the electrical resistivity ρ of a metal.

1.2 Semiconductors

In semiconductors, both electrons and holes contribute to electrical conductivity. Unlike in metals, the charge carrier concentration depends on temperature due to thermal activation. The electrical conductivity σ is given by:

$$\sigma = n e \mu_n + p e \mu_p , \tag{3}$$

the temperature dependence of the conductivity σ can be expressed as a function of the mobilities μ_n und μ_p of electrons and holes, respectively, and the charge carrier concentration, n for electrons and p for holes.

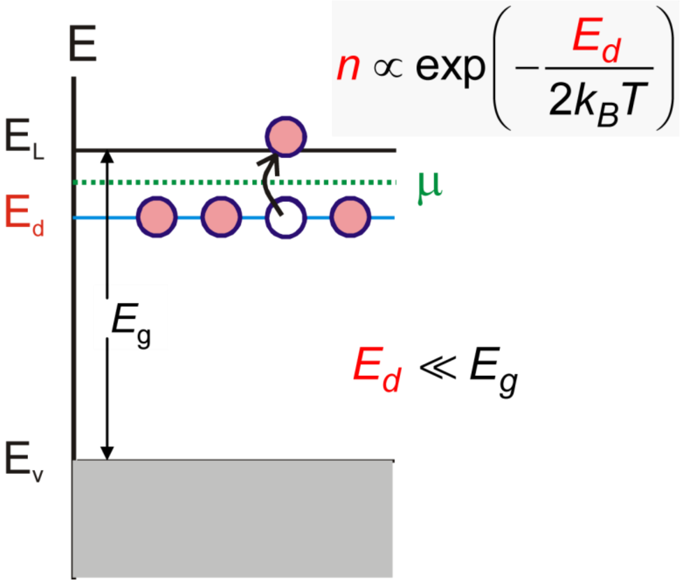

Figure 2: Schematic sketch of electronic bands of an n-doped semiconductor. E_L and E_V denote the edges of the conduction band and the valence band, respectively, E_g the band gap, μ the chemical potential, and E_d the donor level.

1.2.1 Temperature dependence of charge carrier concentration

For n-doped semiconductors, the temperature dependence of charge carrier concentration can be divided into three regimes:

1. *Freeze out* ($E_d \gg k_B T$):
   The thermal energy is not sufficient to ionize all donors from their donor levels E_d; only some electrons are excited into the conduction band:
   $$n \propto e^{\frac{-E_d}{2kT}} . \tag{4a}$$
2. *Extrinsic* ($E_\mathrm{d} \ll k_\mathrm{B} T \ll E_\mathrm{g}$):
   All donors are ionized, but electrons are not thermally excited across the band gap:
   $$n = \text{constant.} \tag{4b}$$
3. *Intrinsic* ($E_\mathrm{g} \ll k_\mathrm{B} T$):
   The thermal energy is sufficient to excite electrons across the band gap E_g and to produce intrinsic charge carriers:
   $$n \propto e^{\frac{-E_g}{2kT}} . \tag{4c}$$

These three regimes are visible on a Arrhenius plot, which shows ln(n) versus 1/T, such as shown in Fig. 3.

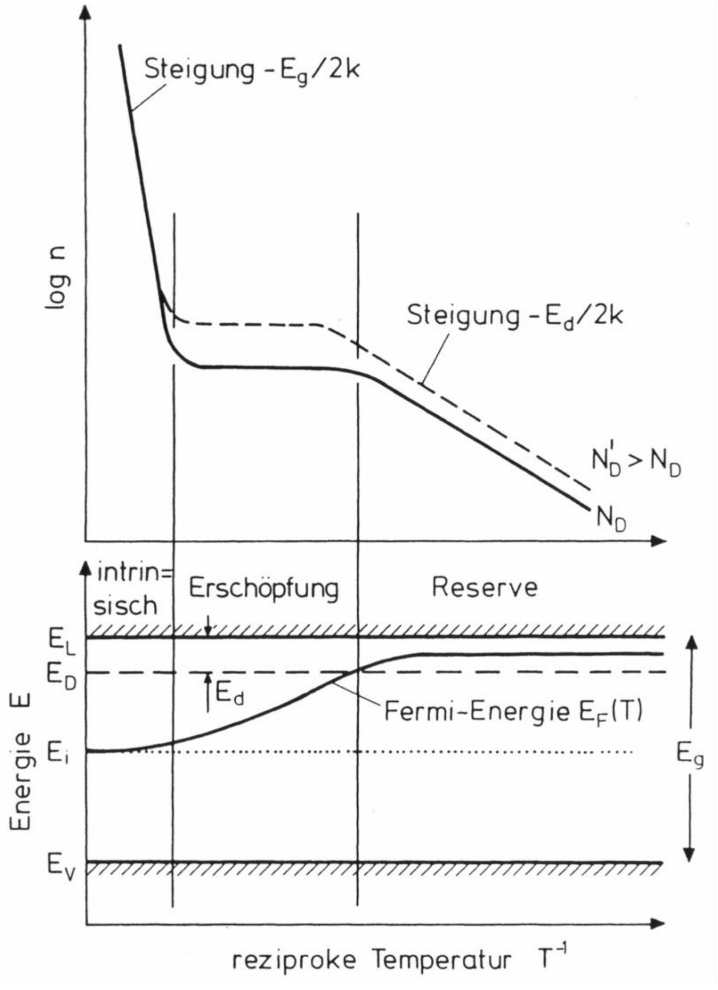

Figure 3:
Temperature dependence of the charge carrier density ("Arrhenius-Plot", top) and of the chemical potential in a doped semiconductor (bottom).

1.2.2 Temperature dependence of the mobility

The mobility of charge carriers also depends on temperature due to scattering mechanisms. Following $\mu(T) = \frac{e\tau}{m}$ for electrons, one can distinguish two temperature regimes:

1) T small: scattering with ionized impurities

$$\mu \propto T^{\frac{3}{2}} \tag{5a}$$

2) T large: scattering with phonons

$$\mu \propto T^{-\frac{3}{2}} \tag{5b}$$

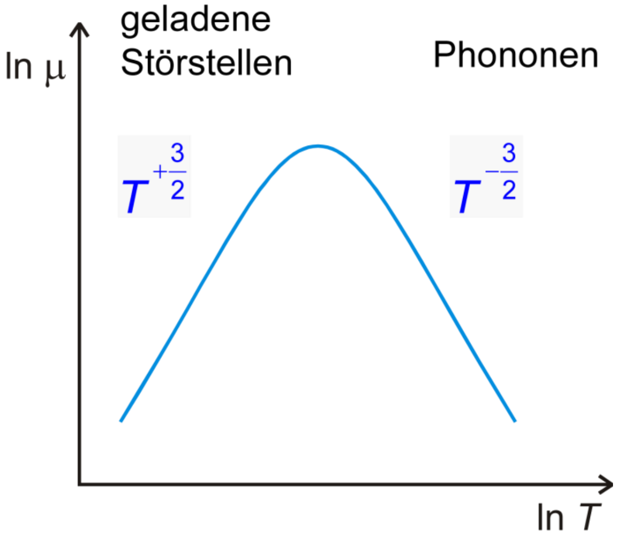

Figure 4:
Temperature dependence of the mobility of a semiconductor.

1.2.3 Temperature dependence of the electrical conductivity

Combining the results for charge carrier concentration (1.2.1) and mobility (1.2.2), we obtain the plot shown in Fig. 5.

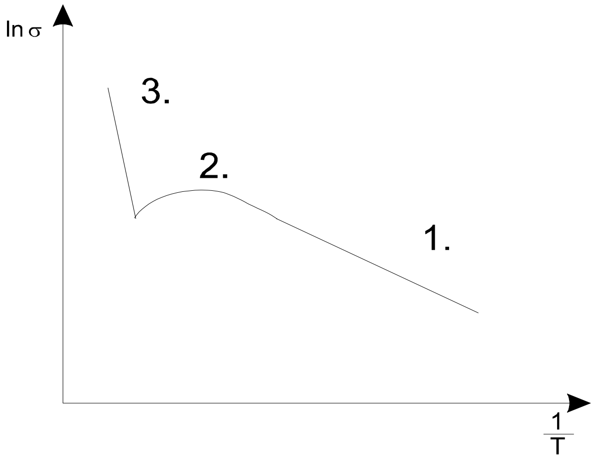

Figure 5:
Temperature dependence of the conductivity of a doped semiconductor.

2. Four-Point-Measurements

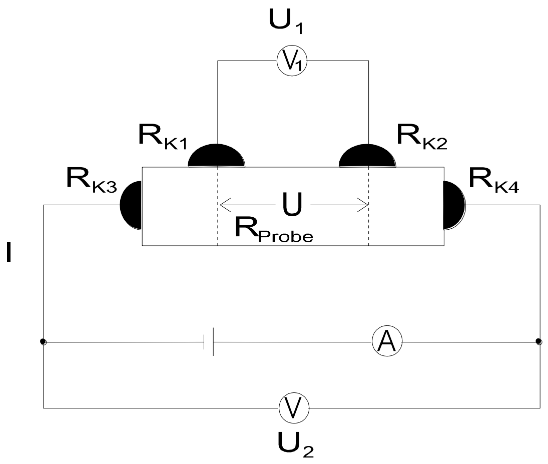

Figure 6a: Four-point-measurement (schematic).

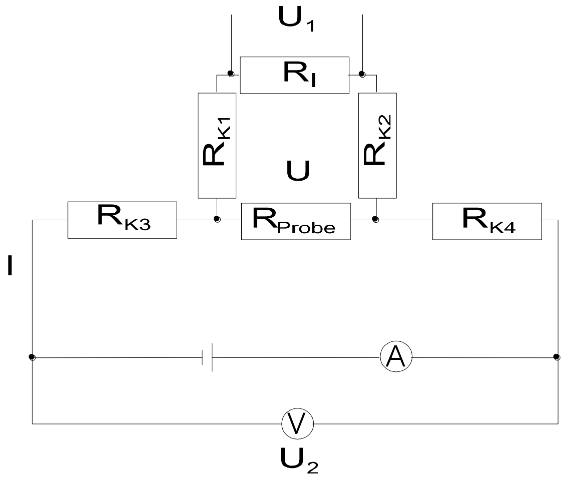

Figure 6b: Corresponding circuit diagram of resistors.

When the resistance of the contacts and wiring is comparable to the sample resistance, a two-terminal measurement introduces significant errors. To address this issue, a four-terminal measurement is preferred, as illustrated in Fig. 6.

Following Kirchhoff's laws, we can write:

$$U = (I - I_{V1})R_{Probe}\,, \tag{6a}$$

$$U = (R_{K1} + R_{K2} + R_I)I_{V1}\,, \tag{6b}$$

$$U_1 = R_I\, I_{V1} \quad \text{(voltage measured by the voltmeter)}, \tag{6c}$$

where R_I denotes the internal resistance of the voltmeter (e.g., $R_I \approx 10\ \text{M}\Omega$ for digital hand-held voltmeter). Comparing (6a) and (6b) gives

$$I_{V1} = \frac{I\, R_{Probe}}{(R_{K1} + R_{K2} + R_I + R_{Probe})}\,. \tag{7}$$

Substitution of (7) in (6c) yields

$$U_1 = R_{Probe}\, I\,(1-\alpha) \tag{8}$$

With

$$\alpha = \frac{R_{K1} + R_{K2} + R_{Probe}}{R_{K1} + R_{K2} + R_{Probe} + R_I} = \left[1 + \frac{R_I}{R_{K1} + R_{K2} + R_{Probe}}\right]^{-1} << 1.$$

As long as $R_I >> R_{Probe}$, R_{K1}, R_{K2}, the measured voltage closely approximates:

$$U_1 \approx I\, R_{Probe}. \qquad (9)$$

Therefore, in a four-terminal measurement, as long as the internal resistance of the voltmeter is significantly larger than the sample and contact resistances, the voltage measured closely approximates the voltage across the sample alone. This eliminates errors due to contact resistances. For comparison, a two-terminal measurement would yield:

$$U_2 = I(R_{K3} + R_{K4} + R_{Probe}), \qquad (10)$$

resulting in a voltage drop dominated by the contact resistances, especially if R_{Probe} is small.

3. Experimental set-up

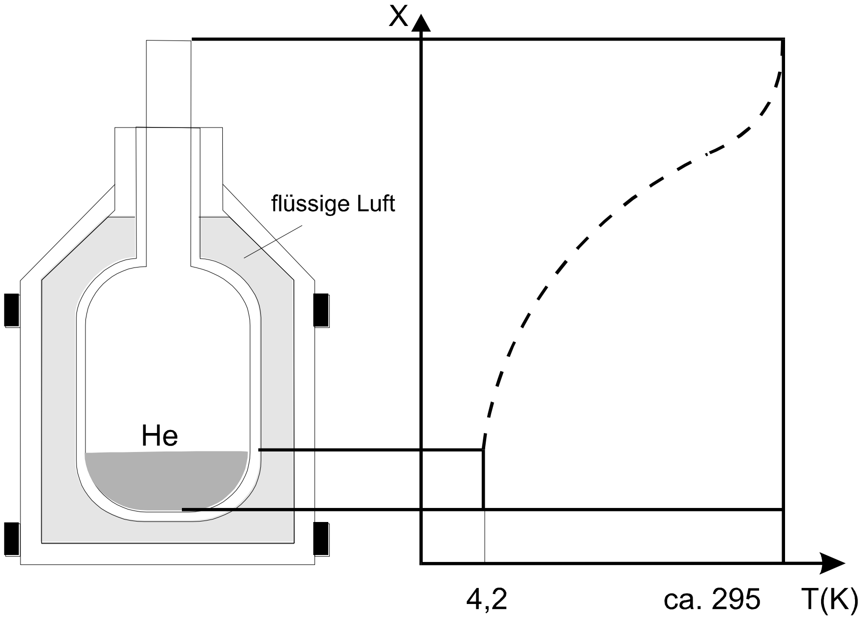

Fig. 7: Experimental set-up.

To vary the temperature, we will make use of vessels containing liquid nitrogen or helium. Within the vessel, a temperature gradient forms along the vertical direction due to heat transfer mechanisms (Fig. 7). The temperature *T*(*x*) depends on the distance *x* from the helium surface. The exact temperature profile is determined by several factors, including the amount of helium, the geometry of the vessel, and its insulation properties.

The samples (Cu, Ta und Si) are mounted on a sample holder (Probenhalter) made out of copper, which is screwed to a rod (Tauchrohr) and covered by a cylindrical shield (Schutzrohr) (Fig. 9). In addition, a platinum and carbon resistor is added to the sample holder, which will be used to measure the temperature.

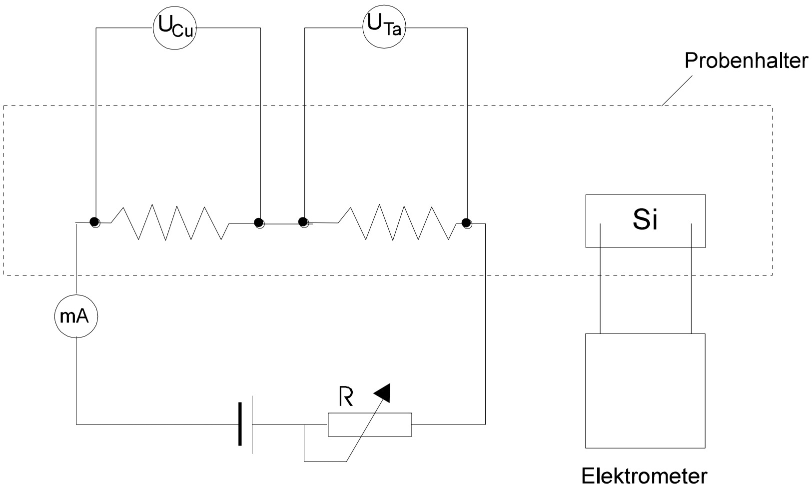

Fig. 8:
Electrical set-up.

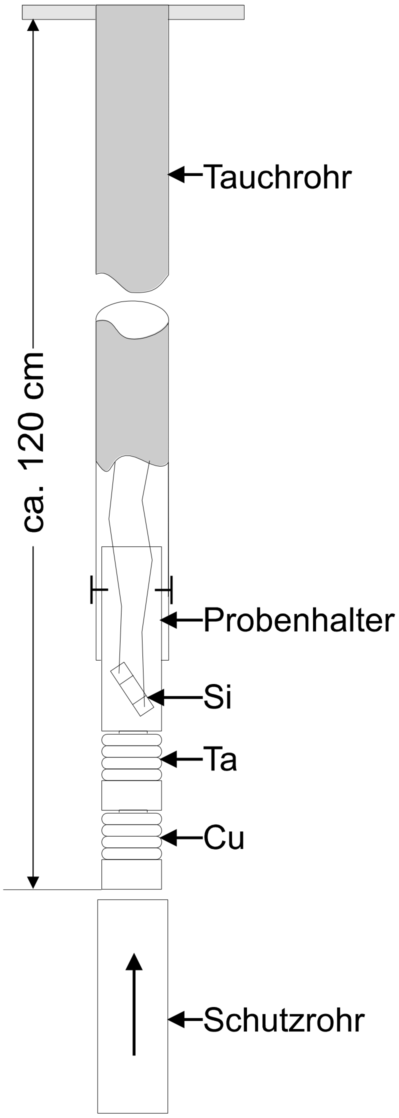

Fig. 9: Sample holder.

3.1 Electrical set-up

The electrical resistance of the Si sample will be measured in a 2-point setup. The coils of Cu and Ta are connected in series, and a constant current of $I = 20$ mA is driven through both of them simultaneously. Two voltage probes are added to each sample, allowing for a four-point measurement of the resistance (see Fig. 8).

The carbon and platinum resistor are measured via a 2-point measurement. During the experiment, a multimeter is used with a switching card. This allows the multimeter to measure the resistances one-by-one. The system is also programmed to automatically switch between 2-point and 4-point measurements. A measurement script switched automatically between the different resistances, and if a button in the software is pressed, the resistance values are appended in a text file.

4. Experimental procedure

The experiment is divided in to two sessions, carried out at different temperatures:

1. 4.2 K to 77.5 K in a helium vessel.
2. 77.5 K to room temperature (RT) in a nitrogen vessel.

Each group has its own probe stick with their samples. One group starts in the morning with helium, and the other with nitrogen. During lunch, the probes will be taken out and allowed to warm back up to room temperature. After lunch, the groups will continue in the other liquid.

1. Using the helium vessel:
   Before inserting it into the helium vessel, the sample rod **has to be at room temperature**. Otherwise, it could freeze to the opening of the vessel and block the exhaust pipe. The sample rod must be inserted **very slowly** since the insulation could be damaged due to large thermal stress. The sample holder has to be inserted into the helium **by the supervisor**. After that, the sample current should be adjusted to $I = 20$ mA. The sample rod is then slowly inserted into the vessel, the temperature equilibrium is waited for and datapoints are recorded.

   Safety note: helium undergoes the largest enthalpy change in the gas form, therefore the cold gas/vapour can easily cause a burn. Should any vapour escape during the experiment, stay clear and call your supervisor (e.g. don’t attempt to plug the leak with your bare hands).

2. Similar to 1.
   Differences: Instead of using liquid helium, liquid nitrogen is used.

   Safety note: compared to helium, the vapour of nitrogen is not so dangerous. However, watch out for any metal parts that can be extremely cold due to the nitrogen, and easily cause a burn if touched.

4.1 Calibration

To analyze your data, you will have to calibrate the platinum and carbon resistors. Luckily, a few temperature points are well known: room temperature (a sensor is available in the lab), the boiling point of liquid nitrogen (77 K) and liquid helium (4,2 K). Platinum is a metal that is well known for its nice linear regime. Therefore, between 77 K and room temperature a simple linear fit is sufficient to accurately calibrate this sensor. The carbon resistor follows an exponential dependency:

$$R_C(T) = a \cdot e^{\frac{b}{T}} + c$$

Therefore, all three measurement points are necessary to calibrate this sensor.

4.2 Assignment

1. Measure the resistance of Cu, Ta and Si as a function of temperature (4.2 to 295 K).
2. Calibrate the platinum and carbon temperature sensors.

5. Evaluation and contents of your report

Your report should contain at least the following:

a) Calibration:
    i. Show how you calibrated your temperature sensors and any validation of the temperature scale that you have performed.

b) Uncertainty analysis:
    i. Discuss your approach on how the uncertainty on your measured resistances and temperatures is determined.

c) For the metals Cu and Ta:
    i. Plot a diagram of $R(T)$ and a diagram of $\ln(R)$ over $\ln(T)$.
    ii. Determine the linear coefficient of resistance $R(T) = R_0\,(1+ \alpha\, T)$ where T in °C and R in Ω have to be used and $R_0 = R(T = 0°\mathrm{C})$.
    iii. Identify the nonlinear regime and determine the exponent β. Does the metal indeed act as a simple metal*?
    * see for example: https://en.wikipedia.org/wiki/Electrical_resistivity_and_conductivity#Temperature_dependence
    iv. For Ta, determine the critical temperature.

d) For the silicon
    i. Make diagrams of:
        i. $R(T)$.
        ii. $\ln(R)$ over $\ln(T)$
        iii. $\ln(R)$ over $1/T$
    ii. Use the diagrams above and determine which regimes are visible in your experiments.
    iii. If possible, extract (or estimate) the size of the bandgap or the ionization energy of the donors. Ignoring the temperature dependence of the mobility if acceptable in this analysis.
    iv. If possible, extract the mobility in the extrinsic regime.

6. Hints/common mistakes and pitfalls

6.1 During the experiment:

1. Determine whether uncertainties in your experiment are systematic or random. Systematic errors (e.g., calibration issues or drift) must not be treated as random.
2. The most reliable method for determining uncertainties is through direct measurement. Design a protocol to measure the uncertainty in your measurement, there are multiple approaches to achieve this.
3. Allow sufficient time for thermalization, however, some residual drift is inevitable. Decide how long to wait, the acceptable level of error, and how to incorporate this drift into your uncertainty analysis.
4. The platinum sensor (pt-100 or pt-1000) is widely used due to its linear resistance-temperature relationship. Use reference tables for resistance vs. temperature values (between 77 K and room temperature) to estimate the temperature and determine how far to adjust the rod. Note that contact resistance introduces slight deviations (a few ohms) from these values.

5. When the probe stick is near the top of the vessel (first ~50 cm), the temperature gradient is minimal. Move the probe stick in larger steps in the beginning of the experiment. As you approach the liquid, where the gradient is larger, take smaller steps.
6. The nitrogen vessel allows for better measurements in the 77K-295K range. Therefore, don't waste time in the helium vessel by taking many data points in that regime, rather focus on the regime below 77 K.

6.2 During the analysis/writing your protocol:

7. Compare any value you extract with reports in literature to strengthen your analysis.
8. Be cautious with the temperature calibration data. The exponential fit can be challenging with only 3 data points. Ensure that both the platinum and carbon temperature scales align. If they do not, add additional data points between 77 K and room temperature.
9. For nonlinear dependencies (e.g., power-law behaviour), use logarithmic plots to clearly demonstrate whether these regimes are present or absent.
10. A convincing analysis is the most important grading criterion. Focus on clear figures and concise presentation of data and results, as large amounts of text are unnecessary.
11. Do not assume your data will perfectly match theoretical expectations. Certain regimes might not be observable within the experimental temperature range (4.2 K–room temperature), or slopes might deviate.
12. Below a certain temperature, the resistance of silicon can become too large for the instrument to measure accurately. When this occurs, the device outputs a placeholder value 98999999999999993400000000000000000000 (9.9E37). Exclude these data points from your analysis, as they do not represent actual resistance values.

6.3 Bonus points:

Bonus points can be earned to offset other mistakes in the report, these can be earned by:

- Including a particularly innovative analysis technique.
- (Correctly) Identifying and analyzing additional phenomena that are not discussed in the instructions.
- Writing with near-publication-level clarity and scientific rigor.

Questions and problems for self-monitoring

1. What determines the electrical resistance:
   a) at room temperature?
   b) at liquid helium temperature?

2. What does Matthiesen's rule say?

3. Using a simple model, explain why in normal metals the resistance is proportional to the temperature.

4. What determines the electrical conductivity of a semiconductor?

5. Sketch the charge carrier concentration of a p- or n-doped semiconductor as a function of temperature.

6. Draw the temperature dependence of the conductivity of a semiconductor.

7. Explain the temperature dependence of the conductivity of a semiconductor, using the energy-band model.

8. Explain the phenomenon superconductivity.

9. What is the use of the four-point-measurement technique?

10. Why is it used for the determination of the resistance of the Cu and Ta samples?