

9 Electrical conductivity

of metals, semiconductors, and superconductors

Basic knowledge:

Boltzmann and Fermi-Dirac-statistics, band structure for metals, undoped and doped semiconductors, basic models of temperature dependence of electrical resistivity in metals and semiconductors, type 1 superconductors, thermocouple, four-point-measurements, electrometer.

Literature:

- [1] Ibach/Lüth: „Festkörperphysik“, Springer-Verlag, Berlin (1981)
Kapitel 5,6,7,8,10, Supraleitung
- [2] Gerthsen: „Physik“
- [3] Ch. Kittel: „Festkörperphysik“, Oldenbourg-Verlag (1988)

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1. Electrical conductivity

1.1 Metals

Starting with a free electron gas and using the Drude model, we obtain the following expression for the electrical conductivity:

$$\sigma = \frac{ne^2\tau}{m^*}, \quad (1)$$

where τ denotes the relaxation time (the average time between two collisions), n the charge carrier density, and m^* the effective electron mass. Because of the Fermi distribution, the charge carrier density is nearly independent of temperature. The only temperature dependence is introduced by the relaxation time τ . There are two main sources of scattering (s. Fig. 1):

- 1) Impurities: Electrons interact with vacancies or impurities, which means that the corresponding resistivity is independent of temperature. At low temperatures, this effect gives rise to a residual resistivity ρ_{ST} .
- 2) Phonons: At higher temperatures, more and more phonons (oscillations of the crystal lattice) are thermally excited. These phonons interact with the conduction electrons. Two temperature regimes are important:

$$T \ll \Theta_D : \quad \rho_{Ph} \propto T^5 \quad (\text{small angle scattering})$$

and

$$T \gg \Theta_D : \quad \rho_{Ph} \propto T,$$

where Θ_D denotes the Debye frequency.

According to the Matthiesen rule, the resistivities introduced by impurities and phonons add up to a total resistivity

$$\rho = \rho_{ST} + \rho_{Ph}. \quad (2)$$

Some metals show superconductivity at low temperatures. Below a critical temperature T_C , the resistivity vanishes. Typical values of T_C are:

Ta:	4,48 K
Pb:	7,20 K
Nb ₃ Sn:	18 K.

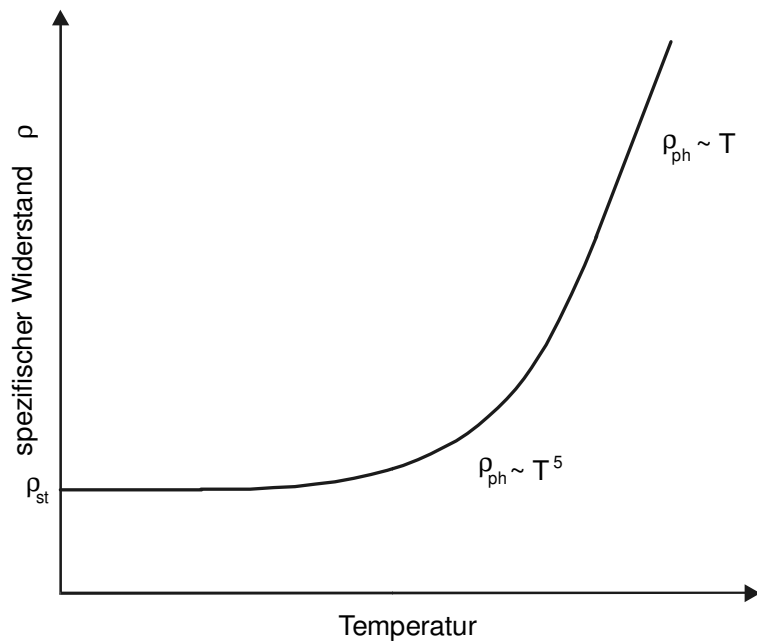


Abb. 1:
Temperature dependence of the electrical resistivity ρ of a metal.

1.2 Semiconductors

In semiconductors, electrons and holes are contributing to the current. Additionally, charge carriers are formed by thermal activation. Their concentration is temperature dependent, in contrary to the case of metals. According to

$$\sigma = n e \mu_n + p e \mu_p, \quad (3)$$

the temperature dependence of the conductivity σ can be expressed as a function of the mobilities μ_n and μ_p of electrons and holes, respectively, and the charge carrier concentration, n for electrons and p for holes.

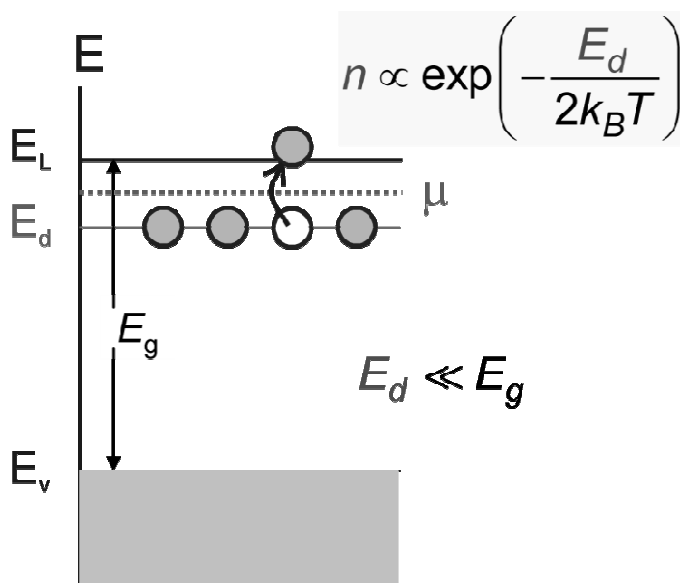


Fig. 2:
Schematic sketch of electronic bands of an n-doped semiconductor. E_L and E_V denote the edges of the conduction band and the valence band, respectively, E_g the band gap, μ the chemical potential, and E_d the donor level.

1.2.1 Temperature dependence of charge carrier concentration

Regarding an n -doped semiconductor, one can distinguish three regimes:

1. Freeze out ($E_d \gg k_B T$):

The thermal energy is not sufficient to ionize all donors, but some are ionized, meaning their electrons are excited from their donor levels E_d into the conduction band:

$$n \propto e^{\frac{-E_d}{2kT}} \quad (4a)$$

2. Extrinsic ($E_d \ll k_B T \ll E_g$):

All donors are ionized, but the thermal energy is not sufficient to excite electrons across the intrinsic band gap:

$$n = \text{const.} \quad (4b)$$

3. Intrinsic ($E_g \ll k_B T$):

The thermal energy is sufficient to excite electrons across the band gap and to produce intrinsic charge carriers. At the same time, the importance of the charge carriers excited from donor levels becomes very small:

$$n \propto e^{\frac{-E_g}{2kT}} \quad (4c)$$

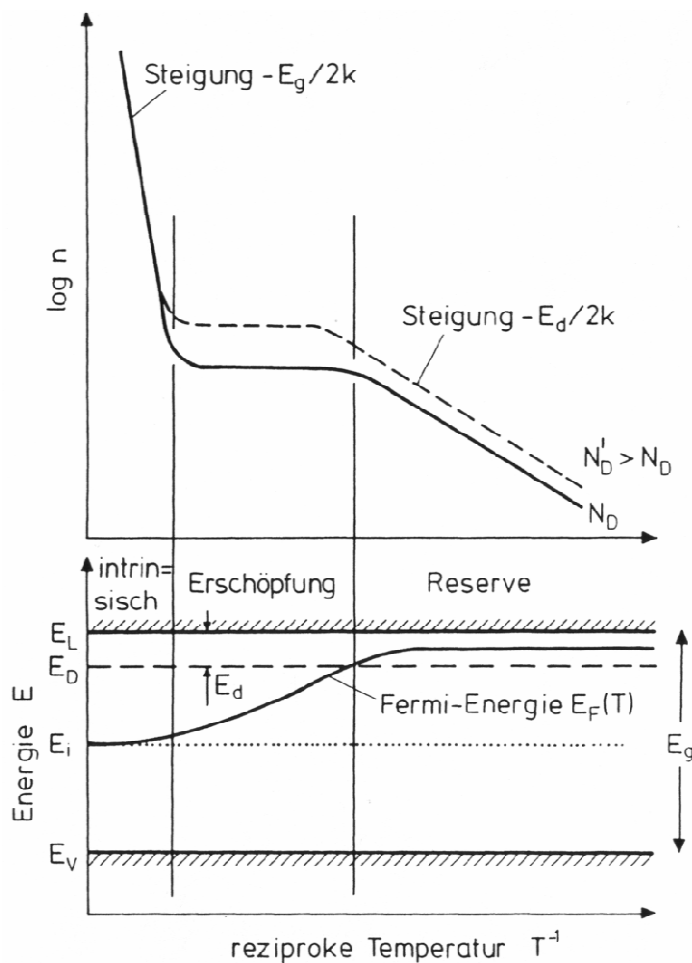


Fig. 3:
Temperature dependence of the charge carrier density ("Arrhenius-Plot", top) and of the chemical potential in a doped semiconductor (bottom).

1.2.2 Temperature dependence of the mobility

Following $\mu(T) = \frac{e\tau}{m}$ for electrons, one can distinguish two temperature regimes:

- 1) T small: scattering with ionized impurities

$$\mu \propto T^{\frac{3}{2}} \quad (5a)$$

- 2) T large: scattering with phonons

$$\mu \propto T^{-\frac{3}{2}} \quad (5b)$$

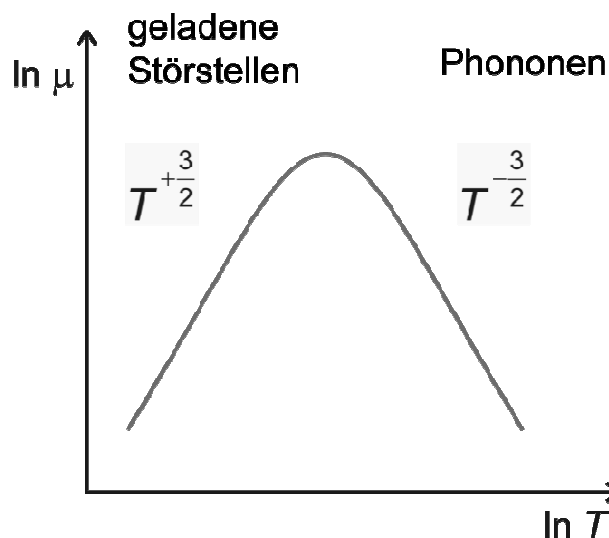


Fig. 4:
Temperature dependence of the mobility of a semi-conductor.

1.2.3 Temperature dependence of the electrical conductivity

Now we combine the relations from 1.2.1 and **Fehler! Verweisquelle konnte nicht gefunden werden.** Since the mobility depends on temperature just following a power law, it has significant influence on the conductivity σ only in the extrinsic regime.

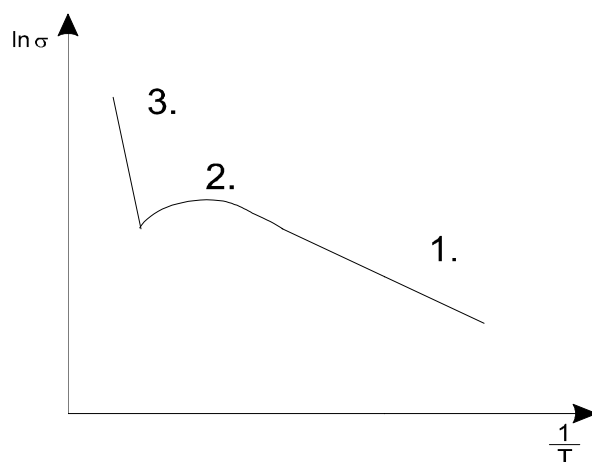


Abb. 5:
Temperature dependence of the conductivity of a doped semiconductor.

2. Four-Point-Measurements

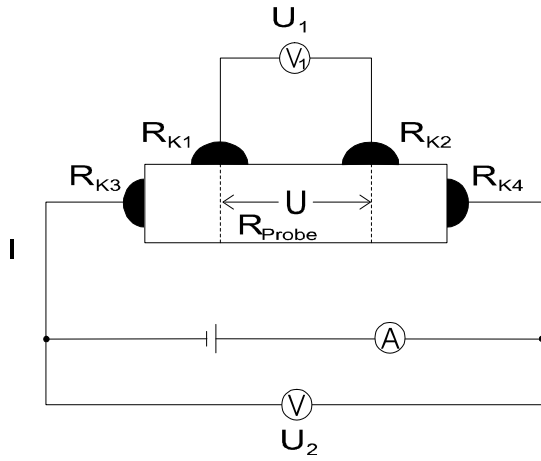


Fig. 6a: Four-point-measurement (schematic).

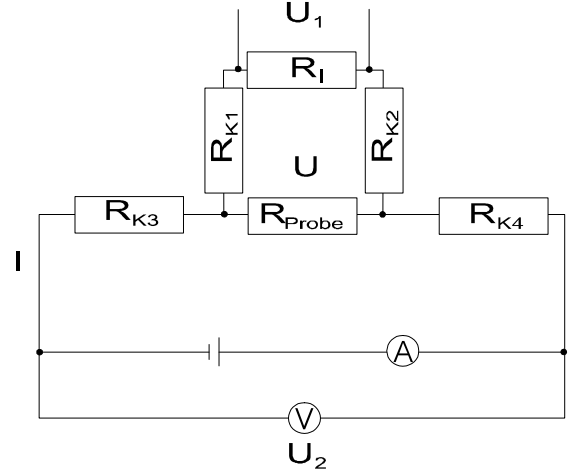


Fig. 6b: Corresponding circuit diagram of resistors.

If the resistance of contacts and wiring are in the same order of magnitude as the sample resistance, a two terminal measurement will produce large errors. A significant improvement is the four-point-measurement, as sketched above.

Following Kirchhoff's circuit laws, we can write:

$$U = (I - I_{V1}) R_{\text{Probe}}, \quad (6a)$$

$$U = (R_{K1} + R_{K2} + R_I) I_{V1}, \quad (6b)$$

$$U_1 = R_I I_{V1} \quad (\text{voltage measured by the voltmeter}), \quad (6c)$$

where R_I denotes the internal resistance of the voltmeter (e.g., $R_I \approx 10 \text{ M}\Omega$ for digital handheld voltmeter). Comparing (6a) and (6b) gives

$$I_{V1} = \frac{I R_{\text{Probe}}}{(R_{K1} + R_{K2} + R_I + R_{\text{Probe}})}. \quad (7)$$

Substitution of (7) in (6c) yields

$$U_1 = R_{\text{Probe}} I (1 - \alpha) \quad (8)$$

With

$$\alpha = \frac{R_{K1} + R_{K2} + R_{\text{Probe}}}{R_{K1} + R_{K2} + R_{\text{Probe}} + R_I} = \left[1 + \frac{R_I}{R_{K1} + R_{K2} + R_{\text{Probe}}} \right]^{-1} \ll 1.$$

Conclusion:

As long as the internal resistance of the voltmeter is large compared to the sample and contact resistances (i.e., almost always), in a very good approximation:

$$U_1 = I R_{\text{Probe}} . \quad (9)$$

That means, only the voltage across the sample is being measured. For comparison: using a two-terminal measurement, one would have obtained:

$$U_2 = I(R_{K3} + R_{K4} + R_{\text{Probe}}) , \quad (10)$$

resulting in a voltage drop dominated by the contact resistances.

3. Experimental set-up

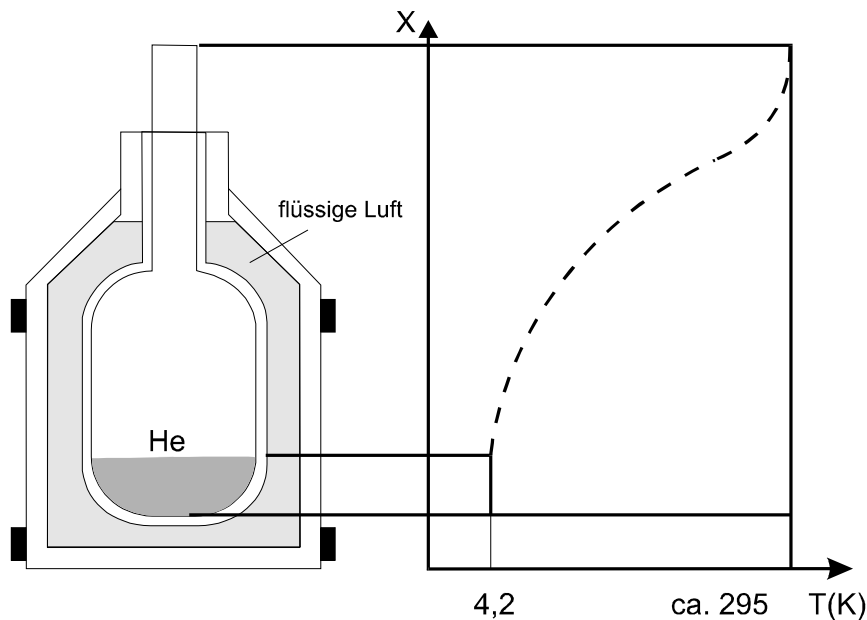


Fig. 7: Experimental set-up.

In a vessel containing liquid helium, a temperature gradient develops. Dependent on the distance x from the He surface, a temperature $T(x)$ is reached. The exact temperature distribution depends on the amount of He and the shape and insulation of the vessel.

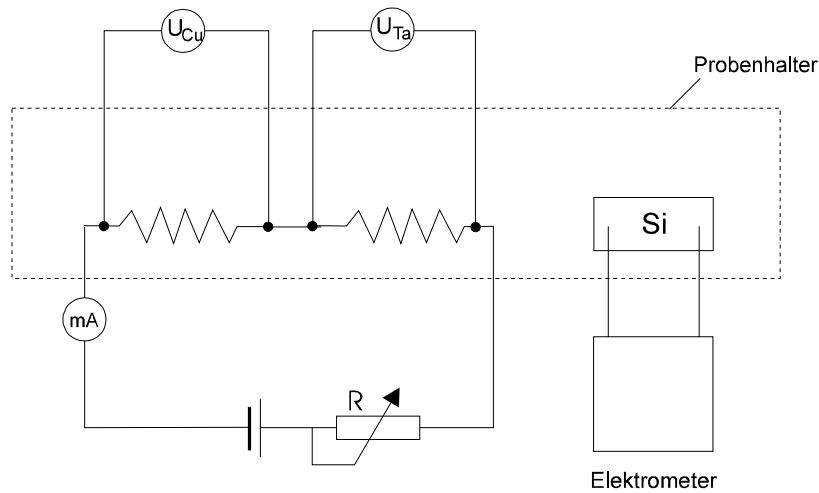


Fig. 8:
Electrical set-up.

The samples (Cu, Ta und Si) are mounted on a sample holder made out of copper, which is screwed to a rod (Tauchrohr) and covered by a cylindrical shield (Schutzrohr) (Fig. 9).

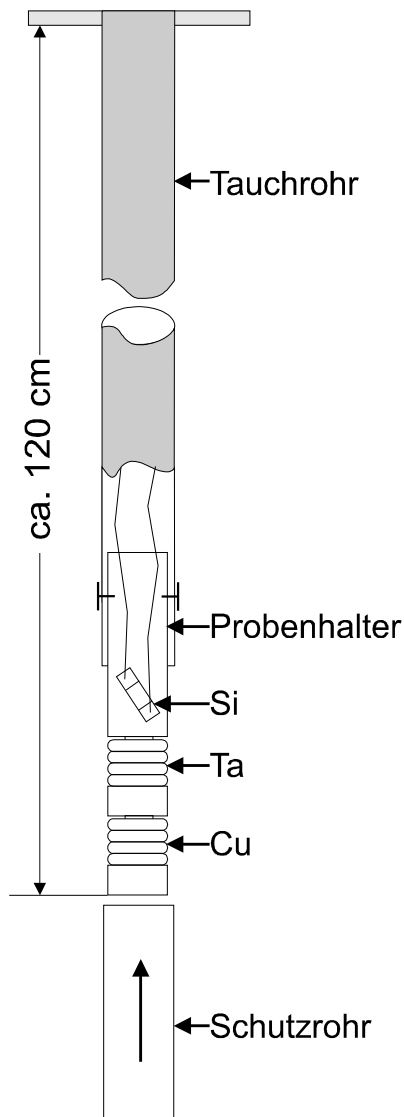


Fig. 9: Sample holder.

3.1 Electrical set-up

The temperature is measured with a platinum resistor at high temperatures and a carbon resistor at low temperatures. Calibration tables for both sample rods are provided (see Tab. 1 and Tab. 2). The resistance varies a little with time due to the thermal stress. Therefore, it is advisable to measure the resistance of the carbon resistors at room temperature before cooling down. The difference of this value compared to the table reading should be subtracted from the measured values at low temperatures.

The electrical resistance of the Si sample will be measured with a resistance bridge or, at low temperatures, directly with an electrometer.

The coils of Cu and Ta are connected in series, and a constant current of $I = 20 \text{ mA}$ is driven through both of them simultaneously. The current value can be adjusted by means of a potentiometer R . The changes of the Cu, Ta, or Nb_3Sn -resistances are negligible compared to R ($R = 500 \Omega$; $R_{\text{Cu}} + R_{\text{Ta}} < 5 \Omega$ for the temperature range covered by the experiment). The voltages U_{Cu} and U_{Ta} are proportional to the resistances R_{Cu} und R_{Ta} which are to be determined.

4. Experimental procedure

The experiment is divided in to two sessions, carried out at different temperatures:

1. 4.2 K to 77.5 K
 2. 77.5 K to room temperature (RT).
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1. Using the helium vessel:
Before inserting it into the helium vessel, the sample rod **has to be at room temperature**. Otherwise, it could freeze to the opening of the vessel and block the exhaust pipe. The sample rod must be inserted **very slowly** since the insulation could be damaged due to large thermal stress. The sample holder has to be inserted into the helium **by the supervisor**. After that, the sample current should be adjusted to $I = 20$ mA. At 4.2 K, U_{Cu} , $U_{\text{Ta, Nb3Sn}}$, and R_{Si} have to be measured. Then the sample rod has to be moved out a little, and one should wait for a new temperature equilibrium.
Der Probenhalter muss sich vor dem Einführen in die Heliumkanne **unbedingt auf Raumtemperatur** befinden, da er sonst am Kannenhals festfrieren und ihn verstopfen kann. Er muss **langsam** eingeführt werden, da durch zu rasches Abkühlen mechanische Spannungen entstehen, die die Isolierung zerstören können. Der **Probenhalter wird vom Betreuer (!)** in das flüssige Helium eingetaucht. Man stelle den Probenstrom $I = 20$ mA ein. Bei 4,2 K werden U_{Cu} , $U_{\text{Ta, Nb3Sn}}$ und R_{Si} notiert. Dann wird das Rohr ein Stück weiter herausgezogen und eine neue Gleichgewichtstemperatur abgewartet.
 2. Similar to 1.
Differences: Instead of using liquid helium, liquid nitrogen is used. R_{Si} is being measured with an electrometer

4.1 Assignment

The electrical resistance of Cu, Ta and Si has to be measured in dependence of temperature (4.2 to 295 K).

5. Evaluation

For Cu, Ta and Si, two diagrams each should be produced:

- a) $R(T)$ and $\ln(R)$ over $\ln(T)$, only in the non-linear regime for Cu and Ta.
- b) $\ln(\frac{1}{R})$ over $\ln(\frac{1}{T})$ and $\ln(\frac{1}{R})$ over $\ln(T)$ in the extrinsic regime (Erschöpfung) for Si.

The following numbers have to be determined:

- c) the linear coefficient of resistance of Cu and Ta, following $R(T) = R_0 (1 + \alpha T)$ where T in $^{\circ}\text{C}$ and R in Ω have to be used and $R_0 = R(T = 0^{\circ}\text{C})$.
- d) for the non-linear regime the exponent β following $R \propto T^{\beta}$ (do not forget the residual resistance!).

e) the critical temperature T_C of Ta or Nb₃Sn, respectively.

f) for the extrinsic regime (Erschöpfung) in Si the mobility $\mu(T)$ following $\frac{1}{R} \propto \sigma \propto \mu(T) \propto T^\gamma$

g) the activation energy of the acceptors, following:

$\sigma(T) \propto p(T) \propto e^{\frac{-(E_A - E_V)}{2k_B T}}$, where $p(T)$ denotes the hole concentration. Neglect the temperature dependence of the mobility.

Questions and problems for self-monitoring

1. What determines the electrical resistance:
 - a) at room temperature?
 - b) at liquid helium temperature?
2. What does Matthiesen's rule say?
3. Using a simple model, explain why in normal metals the resistance is proportional to the temperature.
4. What determines the electrical conductivity of a semiconductor?
5. Sketch the charge carrier concentration of a p- or n-doped semiconductor as a function of temperature.
6. Draw the temperature dependence of the conductivity of a semiconductor.
7. Explain the temperature dependence of the conductivity of a semiconductor, using the energy-band model.
8. Explain the phenomenon superconductivity.
9. What is the use of the four-point-measurement technique?
10. Why is it used for the determination of the resistance of the Cu and Ta (Nb₃Sn) samples?
11. Is thermoelectric voltage also compensated by four-point-measurements?