Ansatz: nicht-relativistische Pauli-Gleichung
(mit ortsfesten Kernen und äußerem Potential $V(\underline{r}) \equiv 0 \ (= \text{const})$)

$$\boxed{i\hbar \frac{\partial}{\partial t} |s\rangle = -\frac{1}{2} g \mu_B \underline{\sigma} \cdot \underline{B} \, |s\rangle} \qquad (*)$$

Allgemeiner Spinzustand:

$$|s\rangle = \underbrace{a_+ e^{-iE_\uparrow t/\hbar}} |\uparrow\rangle + \underbrace{a_- e^{-iE_\downarrow t/\hbar}} |\downarrow\rangle$$

$$= \alpha_+(t) \, |\uparrow\rangle + \alpha_-(t) \, |\downarrow\rangle$$

Einsetzen in $(*)$:

$$i\hbar \frac{\partial}{\partial t} \left[a_+ e^{-iE_\uparrow t/\hbar} |\uparrow\rangle + a_- e^{-iE_\downarrow t/\hbar} |\downarrow\rangle \right]$$

$$= -\frac{1}{2} g \mu_B \underline{\sigma} \cdot \underline{B} \left[\alpha_+(t) |\uparrow\rangle + \alpha_-(t) |\downarrow\rangle \right]$$

$$\Longleftrightarrow \quad E_\uparrow \alpha_+(t) |\uparrow\rangle + E_\downarrow \alpha_-(t) |\downarrow\rangle = -\frac{1}{2} g \mu_B \underline{\sigma} \cdot \underline{B} \left[\alpha_+(t) |\uparrow\rangle + \alpha_-(t) |\downarrow\rangle \right]$$

$| \cdot \langle\uparrow|$ von links

(1) $\Longrightarrow$ $$E_\uparrow \alpha_+(t) = -\frac{1}{2} g \mu_B B_0 \left[\langle\uparrow|\sigma_z|\uparrow\rangle \alpha_+(t) + \langle\uparrow|\sigma_z|\downarrow\rangle \alpha_-(t) \right]$$

$| \cdot \langle\downarrow|$ von links

(2) $$E_\downarrow \alpha_-(t) = -\frac{1}{2} g \mu_B B_0 \left[\langle\downarrow|\sigma_z|\uparrow\rangle \alpha_+(t) + \langle\downarrow|\sigma_z|\downarrow\rangle \alpha_-(t) \right]$$

$\Longleftrightarrow$

(1a) $$E_\uparrow \alpha_+(t) = -\frac{1}{2} g \mu_B B_0 \left[\overbrace{(1,0) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}}^{1} \alpha_+(t) + \overbrace{(1,0) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}^{0} \alpha_-(t) \right]$$

$$E_\downarrow \alpha_-(t) = -\frac{1}{2} g \mu_B B_0 \left[\underbrace{(0,1) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{0} \alpha_+(t) + \underbrace{(0,1) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{-1} \alpha_-(t) \right]$$

$$\Longrightarrow \quad E_\uparrow = -\frac{1}{2} g \mu_B B_0$$

$$E_\downarrow = +\frac{1}{2} g \mu_B B_0$$

$E_\uparrow = -\frac{1}{2} g \mu_N B_0$ mit $\mu_N = \mu_{Proton} = \frac{e\hbar}{2m_p}$

(Kernmagneton)

$E_\downarrow = \frac{1}{2} g \mu_N B_0$

$\Rightarrow E_\uparrow = -\frac{1}{2} g \mu_N B_0$

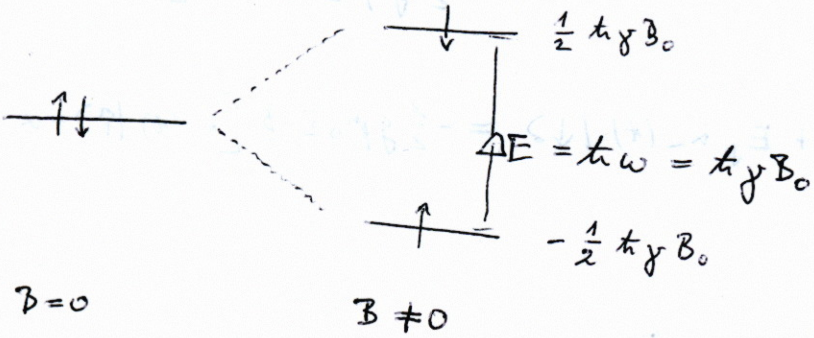
$= -\frac{1}{2} \hbar \underbrace{\frac{g \mu_N}{\hbar}}_{\gamma} B_0$

$\left(\gamma = \frac{g \mu_N}{\hbar} \qquad \mu_N = \frac{e\hbar}{2 m_{Proton}} \right)$

$= -\frac{1}{2} \hbar \gamma B_0$ entsprechend $E_\downarrow = \frac{1}{2} g \mu_N B_0 = \frac{1}{2} \hbar \gamma B_0$

$\Delta E = E_\downarrow - E_\uparrow = \hbar \gamma B_0 \equiv \hbar \omega$

$\Rightarrow \boxed{\omega = \gamma B_0}$



Werte:

$g_{Proton} = 5{,}5857$

$\gamma_{Proton} = 2{,}675 \cdot 10^8 \frac{1}{Ts}$ $\left(\frac{1}{Ts} = \frac{C}{kg} \right)$

$$\langle \hat{S}_z \rangle = \langle \tfrac{\hbar}{2}\hat{\sigma}_z \rangle = \frac{\hbar}{2}\langle s|\hat{\sigma}_z|s\rangle$$

$$= \frac{\hbar}{2}(\alpha_+^*, \alpha_-^*)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix}$$

$$= \frac{\hbar}{2}(\alpha_+^*, \alpha_-^*)\begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix}$$

$$= \frac{\hbar}{2}\left(|\alpha_+|^2 - |\alpha_-|^2\right) \qquad \blacksquare$$

$$\langle \hat{S}_x \rangle = \langle \tfrac{\hbar}{2}\hat{\sigma}_x \rangle = \frac{\hbar}{2}\langle s|\hat{\sigma}_x|s\rangle$$

$$= \frac{\hbar}{2}(\alpha_+^*, \alpha_-^*)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix}$$

$$= \frac{\hbar}{2}(\alpha_+^*, \alpha_-^*)\begin{pmatrix} \alpha_- \\ \alpha_+ \end{pmatrix}$$

$$= \frac{\hbar}{2}(\alpha_+^*\alpha_- + \alpha_-^*\alpha_+)$$

$$= \frac{\hbar}{2}\left(a_+ e^{iE_\uparrow t/\hbar} a_- e^{-iE_\downarrow t/\hbar} + a_- e^{iE_\downarrow t/\hbar} a_+ e^{-iE_\uparrow t/\hbar}\right)$$

$$= \frac{\hbar}{2} a_+ a_- \left(e^{-i\gamma B_0 t} + e^{i\gamma B_0 t}\right)$$

da

$$\left.\begin{aligned} E_\uparrow &= -\tfrac{1}{2}\hbar\gamma B_0 \\ E_\downarrow &= \tfrac{1}{2}\hbar\gamma B_0 \end{aligned}\right\} \Delta E = E_\downarrow - E_\uparrow = \hbar\gamma B_0 \qquad \frac{\Delta E}{\hbar} = \gamma B_0$$

$$= \hbar a_+ a_- \underbrace{\frac{1}{2}\left(e^{-i\gamma B_0 t} + e^{-i\gamma B_0 t}\right)}_{\cos(\gamma B_0 t)}$$

$$= \hbar a_+ a_- \cos(\omega t) \qquad \text{mit } \boxed{\omega = \gamma B_0}$$

$\blacksquare$

$$\langle \hat{S}_y \rangle = \langle \tfrac{\hbar}{2}\hat{\sigma}_y \rangle = \frac{\hbar}{2}\langle s|\hat{\sigma}_y|s\rangle$$

$$= \frac{\hbar}{2}(\alpha_+^*, \alpha_-^*)\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} \alpha_+ \\ \alpha_- \end{pmatrix}$$

$$= \frac{\hbar}{2}(\alpha_+^*, \alpha_-^*)\begin{pmatrix} -i\alpha_- \\ i\alpha_+ \end{pmatrix}$$

$$= -\frac{\hbar\,\overbrace{(i \cdot i)}^{-1}}{2i}(\alpha_+^*\alpha_- - \alpha_-^*\alpha_+)$$

$$= \hbar \cdot \frac{1}{2i}(\alpha_+^*\alpha_- - \alpha_-^*\alpha_+)$$

$$= \hbar \cdot \frac{1}{2i} \left(a_+ a_- \left(e^{-iE_\uparrow t/\hbar} e^{-iE_\downarrow t/\hbar} - e^{iE_\downarrow t/\hbar} e^{-iE_\uparrow t/\hbar} \right)\right)$$

$$= a_+ a_- \hbar \underbrace{\frac{1}{2i} \left(e^{i\gamma B_0 t} - e^{-i\gamma B_0 t} \right)}$$

$$= a_+ a_- \hbar \sin(\gamma B_0 t)$$

$$= a_+ a_- \hbar \sin(\omega t)$$

$\blacksquare$